

RADAR EQUATION

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RADAR is an acronym that stands for "RADIO Detection And Ranging." This is an *active* remote sensing technique because it involves a transmitter sending out pulses of electromagnetic radiation, then measuring the amount of power reflected (scattered) back to the radar antenna. This process can be quantified in the Radar Equation.

The Radar Equation is given as follows:

$$p_r = \frac{\pi^3 p_t g^2 \theta \phi h |K|^2 l z}{1024 \ln(2) \lambda^2 r^2}$$

where

p_t = power transmitted by radar (watts)

p_r = power received back by radar (watts)

g = gain of the antenna (ratio of power on the beam axis to power from an isotropic [i.e., radiating equally in all directions] antenna at the same point); it is a measure of how focused the radar beam is.

θ = horizontal beamwidth (radians)

ϕ = vertical beamwidth (radians)

h = pulselength (m)

$|K|^2$ = dielectric constant for hydrometeors; usually taken as 0.93 for liquid water, 0.197 for ice. (Note that for an equivalent mass of frozen precipitation, much less power is returned from the ice than from liquid precipitation; thus snow with the same water content is less reflective than rain). For this reason, NEXRAD's clear air mode rather than precipitation mode is sometimes used to monitor snow situations because of its greater sensitivity).

l = loss factor for attenuation of radar beam, varies between 0 and 1, usually near 1. Since the attenuation of the beam is often unknown, it is often ignored.

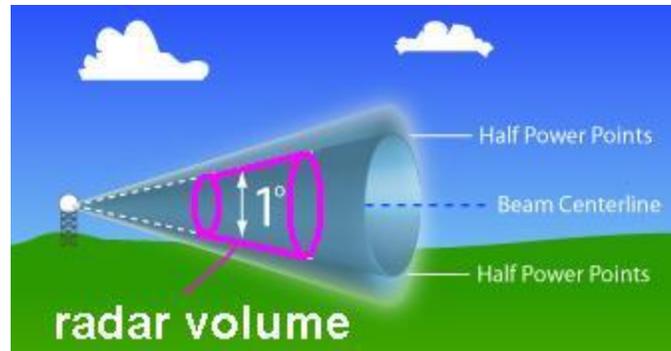
λ = wavelength of radar pulse (m)

r = range or distance to the target (i.e., the distance to an area of precipitation that reflects the originally transmitted pulse back to the radar).

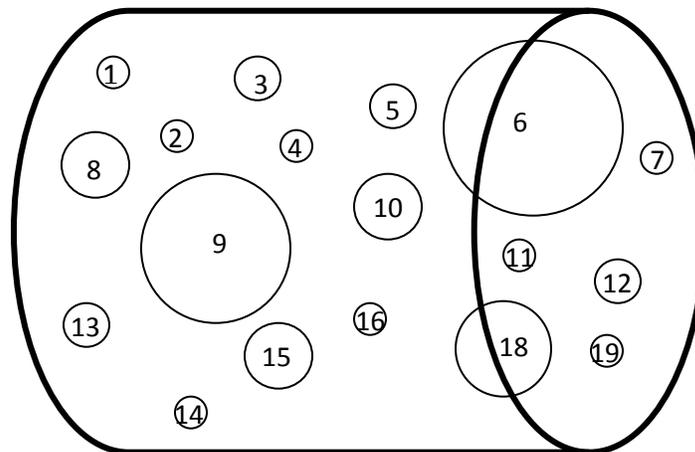
z = radar reflectivity factor (mm^6/m^3) and can be expressed as

$$z = \sum_{vol} D^6$$

where D is the drop diameter and the summation is over the total number of drops (of varying sizes) within a *unit volume* within the beam; in the equation it gets multiplied by the radar volume [defined by the beam width, height, pulse length and distance from the radar]



What does this mean? To illustrate, consider, a 1 cubic meter section of the radar beam containing a distribution of different-sized raindrops, say, 19 of them, each identified with a number from 1 through 19:



The summation for radar reflectivity factor, z , could be expanded as follows:

$$z = D_1^6 + D_2^6 + D_3^6 + \dots + D_{18}^6 + D_{19}^6$$

That is, z is a function of the diameter of drop 1 to the sixth power, plus the diameter of drop 2 to the sixth power and so on, that is, the *drop size distribution*.

Note that z is an inherent property of the drop size distribution sampled and is not radar-dependent.

In other words, it is a property of the precipitation characteristics within a given storm, and whatever drop sizes happen to be in that storm, and has nothing to do with the radar.

Another important point is that because z is proportional to the 6th power of the drop sizes, larger drops make for ***much more*** reflectivity than small drops.

[Note: The radar equation can also be written in this slightly different form

$$p_r = \frac{\pi^3 p_t g^2 \theta \varphi c t |K|^2 l z}{1024 \ln(2) \lambda^2 r^2}$$

where h , the pulse length in distance units, is replaced by c , the speed of electromagnetic radiation, and t , the pulse duration.]

However, the drop size distribution in the measured volume is unknown. Therefore, we calculate the radar reflectivity factor, z , from the returned power, p_r , by solving the above equation for z :

$$z = \frac{p_r 1024 \ln(2) \lambda^2 r^2}{\pi^3 p_t g^2 \theta \varphi c t |K|^2 l}$$

We can combine the known (radar-specific) variables like beamwidth, gain etc., the numerical values, and the assumed

values ($|K|^2$ is assumed as the liquid water value, 0.93, since we don't know *a priori* what the precipitation type is, even though it may actually be snow or hail, and l is usually assumed to be 1) in the above equation into a single known constant, c_1 , to arrive at the simplified expression:

$$z = c_1 p_r r^2$$

Given a specific radar and configuration, the only real independent variables in this problem are the amount of power returned, p_r , and the range, r , to the echoes, which are both measured by the radar. Thus, the radar reflectivity factor, z , can be calculated from those known items quantified in c_1 combined with those two things that the radar is measuring, p_r , and r .

Because the radar reflectivity factor spans a huge range of magnitudes (from 0.001 mm^6/m^3 for fog, to 36,000,000 mm^6/m^3 for softball-sized hail, it is usually expressed in decibels (dB) of reflectivity or dBZ as follows:

$$Z = 10\log_{10}(z / 1 \text{ mm}^6/\text{m}^3)$$

[Note: be sure to distinguish between capital Z and lower case z here!]

The logarithmic transformation here is used to compress the large range of magnitudes into a more comprehensible scale of values. Logarithms are actually just exponents, so the "log₁₀ of z " is just the exponent that 10 would be raised to, to obtain a value of z .

The following table shows interrelationships between z , Z , exponents, logs, and the decibel scale:

z Radar reflectivity factor from the radar equation. (linear scale of reflectivity)	$10^x = z$	$x = \log_{10} z$	Z dBZ = $10 \log_{10} z$ (decibel scale of reflectivity)
0.001	10^{-3}	-3	-30
0.01	10^{-2}	-2	-20
0.1	10^{-1}	-1	-10
1	10^0	0	0
10	10^1	1	10
100	10^2	2	20
1,000	10^3	3	30
10,000	10^4	4	40
100,000	10^5	5	50
1,000,000	10^6	6	60
10,000,000	10^7	7	70

Color-filled contours of 22 levels of reflectivity, Z, are what is plotted on [NEXRAD radar data displays](#). Prior to NEXRAD, reflectivity from the WSR-57's or WSR74's was plotted as 6 levels of reflectivity called VIP (Video Integrator Processor) levels. NEXRAD data plotted as VIP levels can be found on an Aviation Digital Data Service radar page showing [radar coded messages](#) .

Rain rates from Reflectivity

Rain rates can be calculated from radar reflectivity data if we assume that we know the drop size distribution within a volume of air measured by the radar. So-called z-R relationships relate the radar reflectivity factor, z (mm^6/m^3), to the rain rate, R (mm/hr) based on different assumed drop sizes for different kinds of storms, and are of the form

$$z = A \cdot R^b$$

where A and b are empirically-derived constants. Over the years, many z-R relations have been obtained by researchers. A few z-R relations recommended by the Radar Operations Center are listed in this table borrowed from

http://www.ou.edu/radar/z_r_relationships.pdf

Relationship	Optimum for:	Also recommended for:
Marshall-Palmer ($z = 200R^{1.6}$)	General stratiform precipitation	
East-Cool Stratiform ($z = 130R^{2.0}$)	Winter stratiform precipitation - east of continental divide	Orographic rain -East
West-Cool Stratiform ($z = 75R^{2.0}$)	Winter stratiform precipitation - west of continental divide	Orographic rain -West
WSR-88D Convective ($z = 300R^{1.4}$)	Summer deep convection	Other non-tropical convection
Rosenfeld Tropical ($z = 250R^{1.2}$)	Tropical convective systems	

Note that this form of the z-R relation is not useful for calculating the rain rate—we must solve the equation for R and convert from radar reflectivity factor, z, to reflectivity, Z, then we can plug in the reflectivity measured by the radar.

The result for a Marshall-Palmer drop size distribution is:

$$RR = C 10^{(0.0625 Z)}$$

where RR is rain rate in mm/hr, $C = 0.036$ mm/hr, and Z is the reflectivity in dBZ.

Example: For a reflectivity of 39 dBZ, the rain rate is

$$\begin{aligned} RR &= 0.036 \times 10^{(0.0625 \times 39)} \\ &= 9.86 \text{ mm/hr} \times (1 \text{ cm}/10 \text{ mm}) \times (1 \text{ in}/2.54 \text{ cm}) \\ &= 0.39 \text{ inches/hour} \end{aligned}$$